Calc BC - Vectors "Day 6" Homework (3)

① a) magnitude $= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\Big|_{t=2} \Rightarrow \sqrt{(2t)^2 + (2t^2)^2}\Big|_{t=2}$

$= \sqrt{16+64} = \sqrt{80}$

b) Distance $= \int_0^4 \sqrt{(2t)^2 + (2t^2)^2}\, dt$

c) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t^2}{2t} = t$ since $x = t^2 - 2$

$t = \sqrt{x+2}$

$\frac{dy}{dx} = \sqrt{x+2}$

d) on y-axis $\Rightarrow x(t) = 0 \Rightarrow t^2 - 2 = 0 \Rightarrow t = \sqrt{2}$

$a(t) = \langle 2, 4t \rangle \quad a(\sqrt{2}) = \langle 2, 4\sqrt{2} \rangle$

② a) $x(t) = \int \frac{1}{t+1}\, dt \qquad y(t) = \int 2t\, dt$

$x = \ln|t+1| + c \qquad y = t^2 + c$

$\ln 2 = \ln|2| + c \qquad 4 = 1^2 + c$

$c = 0 \qquad c = +3$

Position vector $= \langle \ln|t+1|, t^2 + 3 \rangle$

b) $t = 1$, $(\ln 2, 4)$ $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t}{\frac{1}{t+1}}$ $\left.\frac{dy}{dx}\right|_{t=1} = 4$

$y - 4 = 4(x - \ln 2)$

$\Rightarrow$

② c) Magnitude $= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\Big|_{t=1} = \sqrt{\left(\frac{1}{t+1}\right)^2 + (2t)^2}\Big|_{t=1}$

$= \sqrt{4\frac{1}{4}} = \frac{\sqrt{17}}{2}$

d) $\frac{dy}{dx} = \frac{2t}{\frac{1}{t+1}} \cdot 2t(t+1) = 2t^2 + 2t \; ; \; 2t^2 + 2t = 12$

$2t^2 + 2t - 12 = 0$

$2(t^2 + t - 6) = 0$

$2(t+3)(t-2) = 0$

~~$t = -3$~~ $\quad t = 2$

③ $t^2 + 2\cos t = 7 \Rightarrow t = 2.996$

$v(t) = \langle 2 + 3\cos t, \; 2t - 2\sin t \rangle \; ; \; v(t)\Big|_{t=2.996} = \langle -0.968, \; 5.704 \rangle$

④ $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \Rightarrow t + 3 = \frac{\frac{dy}{dt}}{1 + \sin(t^3)}$

$\frac{dy}{dt} = (t+3)(1 + \sin(t^3))$

$a(t) = \langle 3t^2 \cdot \cos(t^3), \; 1 + \sin(t^3) + 3t^2 \cdot \cos(t^3)(t+3) \rangle$

$a(2) = \langle -1.746, \; -6.741 \rangle$

⑤ a) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \Rightarrow \left.\frac{dy}{dx}\right|_{t=1} = -0.451$

$y - 2 = -0.451(x-3)$

b) Speed $= \sqrt{(\cos(e^t))^2 + (\sin(e^t))^2} = 1$

c) Distance $= \int_0^2 \sqrt{(\cos(e^t))^2 + (\sin(e^t))^2}\, dt = 2$

d) $x(2) = 3 + \int_1^2 \left(\frac{dx}{dt}\right) dt = 2.896$

$y(2) = 2 + \int_1^2 \left(\frac{dy}{dt}\right) dt = 1.676$

Position: $(2.896, 1.676)$

⑥ a) $a(3) = \langle 11.029, 23.545 \rangle$ (on calc)

b) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ $\left.\frac{dy}{dx}\right|_{t=3} = \frac{\cos(24)}{\sin(24)} = 0.468$ $y - 4 = -0.468(x-1)$

c) Magnitude $= \left.\sqrt{(\sin(t^3-t))^2 + (\cos(t^3-t))^2}\right|_{t=3} = 1$

d) $x(t) = 1 - \int_2^3 \sin(t^3-t)\, dt = 0.932$

$y(t) = 4 - \int_2^3 \cos(t^3-t)\, dt = 4.002$

Be careful! moving backwards

Position: $(0.932, 4.002)$

(7)

a) $y(1) = 5 - \int_1^3 (2 + \sin(e^t)) \, dt$

$= 1.269$

b) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2 + \sin(e^t)}{\frac{dx}{dt}} = -1.8$

$\frac{dx}{dt} = \frac{2 + \sin(e^3)}{-1.8}$

$= -1.636$

c) speed $= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \Big|_{t=3}$

$= \sqrt{\left(\frac{2 + \sin(e^3)}{-1.8}\right)^2 + (2 + \sin(e^3))^2}$

$= 3.368$